Predictions of SUSY Flavor Models

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Where Do We Stand?

- Latest 3 neutrino global analysis including atm, solar, reactor, LBL (T2K/MINOS) experiments:

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

$$P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

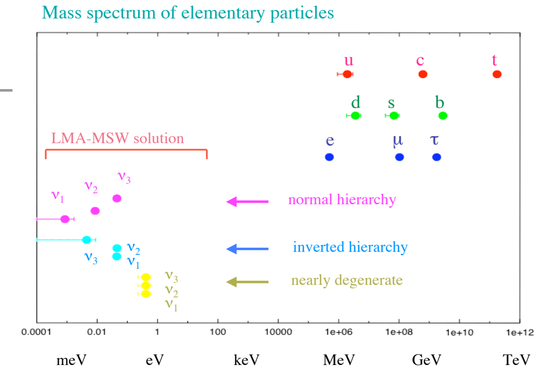
Current Global Fit: $\theta_{13} \neq 0$ at 3σ

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.58	0.306	0.021	0.42	2.35
		(0.312)	(0.025)		
1σ range	7.32 – 7.80	0.291 – 0.324	0.013 – 0.028	0.39 – 0.50	2.26 – 2.47
		(0.296 – 0.329)	(0.018 – 0.032)		
2σ range	7.16 – 7.99	0.275 – 0.342	0.008 – 0.036	0.36 – 0.60	2.17 – 2.57
		(0.280 – 0.347)	(0.012 – 0.041)		
3σ range	6.99 – 8.18	0.259 – 0.359	0.001 – 0.044	0.34 – 0.64	2.06 – 2.67
		(0.265 – 0.364)	(0.005 – 0.050)		

Cautions!! Different global fit analyses assume different error correlations among experiments ⇒ different results

Hint of $\theta_{13} \neq 0$ is exciting, but too early to say how big it really is.

Origin of Mass Hierarchy and Mixing



- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim ⇒ reduce the # of parameters by imposing symmetries
- SUSY ⇔ Flavor?
  - SUSY: allow observable processes probing flavor structure
  - Flavor Symmetry: allow possible determination of sparticle spectrum
- Two examples:
  - SUSY SU(5) x T′ Model M.-C.C, K.T. Mahanthappa, Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009); arXiv:1107.3856 [hep-ph]; M.-C.C. Mahanthappa, Meroni, Petcov, under preparation
  - non-anomalous U(1)′ Family Symmetry in AMSB M.-C. C., J.-R. Huang, arXiv:1011.0407

Tri-bimaximal Neutrino Mixing

- Neutrino Oscillation Parameters $P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t \rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Latest Global Fit (3σ)

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

$$\sin^2\theta_{atm} = 0.42\ (0.34 - 0.64)\ ,\ \sin^2\theta_{\odot} = 0.306\ (0.259 - 0.359)$$

$$\sin^2\theta_{13} = 0.021\ (0.001 - 0.044)$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

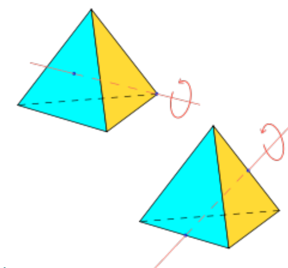
$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

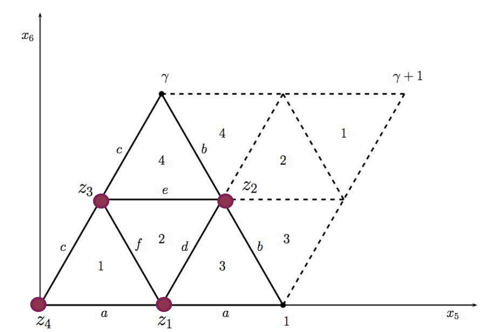
Double Tetrahedral T´ Symmetry

- Smallest Symmetry to realize TBM ⇒ Tetrahedral group A_4 Ma, Rajasekaran (2004)
  - even permutations of 4 objects

    S: (1234) → (4321),   T: (1234) → (2314)
  - invariance group of tetrahedron
  - can arise from extra dimensions: 6D → 4D Altarelli, Feruglio (2006)
  - does NOT give quark mixing



- Double Tetrahedral Group T´ Frampton, Kephart (1995); Aranda, Carone, Lebed (2000); M.-C.C., K.T. Mahanthappa PLB652, 34 (2007); 681, 444 (2009)
  - inequivalent representations



A4: $1,\ 1',\ 1'',\ 3$ (vectorial) → TBM for neutrinos

other: $2,\ 2',\ 2''$ (spinorial) → 2 +1 assignments for charged fermions

  - complex CG coefficients when spinorial representations are involved

Group Theory of T´

- intrinsic complex CG coefficients in T′ (complexity independent of choice of basis for generators)
- spinorial x spinorial ⊃ vector:

J. Q. Chen & P. D. Fan, J. Math Phys 39, 5519 (1998)

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1$$

$$3 = \begin{pmatrix} \left(\frac{1-i}{2}\right)(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

- spinorial x vector ⊃ spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2''$$

$$2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$

Novel Origin of CP Violation

M.-C.C, K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>
- Complex CG coefficients in T′ ⇒ explicit CP violation
  - real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - no additional parameters needed ⇒ extremely predictive model!

The Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009); arXiv:1107.3856 [hep-ph]

- Symmetry: SUSY SU(5) x T′
- Particle Content $10(Q,\, u^c,\, e^c)_L$ $\quad \overline{5}(d^c, \ell)_L$ $\quad \omega = e^{i\pi/6}.$

	T_3	T_a	$\overline{F}$	N	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	ζ'	ξ	η	S
SU(5)	10	10	$\overline{5}$	1	5	$\overline{5}$	45	1	1	1	1	1	1	1	1	1
T'	1	2	3	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1	1
Z_{12}	ω^5	ω^2	ω^5	ω^7	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^5	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1	ω^2

- additional $Z_{12} \times Z'_{12}$ symmetry:
  - predictive model: only 10 operators allowed up to at least dim-7
  - vacuum misalignment: neutrino sector vs charged fermion sector
  - mass hierarchy: lighter generation masses allowed only at higher dim
  - forbids Higgsino mediated proton decay

The Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009);
arXiv:1107.3856 [hep-ph]

- Symmetry: SUSY SU(5) x T´ x Z_{12} x Z_{12}

	SU(5)	T′	
$10(Q,\, u^c,\, e^c)_L$	:	$(T_1, T_2) \sim 2$, $T_3 \sim 1$	1: (N1,N2,N3) ~ 3
$\overline{5}(d^c, \ell)_L$	:	$(F_1, F_2, F_3) \sim 3$	

- Superpotential: only 10 operators allowed

(7+2) parameters fit 22 masses, mixing angles, CPV measures

spinorial representations ⇒ complex CGs

⇒ CPV in quark & charged lepton sector

$$\mathcal{W}_{\rm Yuk} = \mathcal{W}_{TT} + \mathcal{W}_{TF} + \mathcal{W}_\nu$$

$$\mathcal{W}_{TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} H_5 \left[y_{ts} T_3 T_a \psi\zeta + y_c T_a T_b \phi^2 \right] + \frac{1}{\Lambda^3} y_u H_5 T_a T_b \phi'^3$$

up type quarks

$$\mathcal{W}_{TF} = \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi\zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi\psi\zeta' + y_d H_{\overline{5}'} \overline{F} T_a \phi^2 \psi' \right]$$

down type quarks & charged leptons

$$\mathcal{W}_\nu = \lambda_1 NNS + \frac{1}{\Lambda^3}\left[H_5 \overline{F} N \zeta\zeta' \left(\lambda_2 \xi + \lambda_3 \eta \right) \right]$$

neutrino masses

Λ : scale above which T' is exact

Reality of Yukawa couplings: ensured by degrees of freedom in field redefinition

Neutrino Sector

- Operators:

$$\mathcal{W}_\nu = \lambda_1 NNS + \frac{1}{\Lambda^3}\left[H_5\overline{F}N\zeta\zeta'\left(\lambda_2\xi + \lambda_3\eta\right)\right]$$

alternative seesaw: leptogenesis
Dirac mass terms with dim-7
-- low RH neutrino masses
-- flavor effect important

- symmetry breaking

$$T' \to G_{TST^2}: \qquad \langle\xi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}\xi_0\Lambda \qquad T'-\text{invariant:} \quad \langle\eta\rangle = \eta_0\Lambda \qquad \langle S\rangle = S_0$$

- resulting mass matrices

$$M_{RR} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{pmatrix}S_0 \qquad M_D = \begin{pmatrix}2\xi_0+\eta_0 & -\xi_0 & -\xi_0\\-\xi_0 & 2\xi_0 & -\xi_0+\eta_0\\-\xi_0 & -\xi_0+\eta_0 & 2\xi_0\end{pmatrix}\zeta_0\zeta_0' v_u$$

only vector representations
⇒ all CG are real
⇒ Majorana phases: 0 or π

- seesaw mechanism: effective neutrino mass matrix

$$U_{TBM}^T M_\nu U_{TBM} = \mathrm{diag}((3\xi_0+\eta_0)^2, \eta_0^2, -(-3\xi_0+\eta_0)^2)\frac{(\zeta_0\zeta_0' v_u)^2}{S_0} \qquad U_{\mathrm{TBM}} = \begin{pmatrix}\sqrt{2/3} & 1/\sqrt{3} & 0\\-\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2}\\-\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2}\end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

General conditions for form diagonalizability:
M.-C.C., S.F. King, JHEP0906, 072 (2009)

Up Quark Sector

- Operators:

$$\mathcal{W}_{TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} H_5 \left[y_{ts} T_3 T_a \psi \zeta + y_c T_a T_b \phi^2 \right] + \frac{1}{\Lambda^3} y_u H_5 T_a T_b \phi'^3$$

- top mass: allowed by T′
  - lighter family acquire masses thru operators with higher dimensionality
  - dynamical origin of mass hierarchy
- symmetry breaking:

$$T' \to G_T \qquad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \, , \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda \qquad T' \to G_S : \quad \langle \zeta \rangle = \zeta_0$$

dim-6

no contributions to elements involving 1st family; true to all levels

$$T' \to G_{TST^2} : \qquad \langle \phi' \rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

dim-7

- Mass matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

both vector and spinorial reps involved
⇒ complex CG

Down Quark & Charged Lepton Sectors

- operators: $$\mathcal{W}_{TF} = \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi \psi \zeta' + y_d H_{\overline{5}'} \overline{F} T_a \phi^2 \psi' \right]$$
- generation of b-quark mass: breaking of T', dynamical origin for hierarchy between m_b and m_t
- lighter family acquire masses thru operators with higher dimensionality
- symmetry breaking:

$$T' \to G_T: \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \,, \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda$$

$$T' \to \text{nothing:} \quad \langle \psi' \rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T' \to G_S: \quad \langle \zeta \rangle = \zeta_0 \,, \langle \zeta' \rangle = \zeta'_0$$

- mass matrix:

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0 \psi'_0 & 0 \\ -(1-i)\phi_0 \psi'_0 & \psi_0 \zeta'_0 & 0 \\ \phi_0 \psi'_0 & \phi_0 \psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \qquad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0 \psi'_0 & \phi_0 \psi'_0 \\ (1+i)\phi_0 \psi'_0 & -3\psi_0 \zeta'_0 & \phi_0 \psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

- consider 2nd, 3rd families only: TBM exact

complex CG

- Georgi-Jarlskog relations: $m_d \simeq 3 m_e \qquad m_\mu \simeq 3 m_s \quad \longrightarrow$ corrections to TBM

Model Predictions

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009); arXiv:1011.3856 [hep-ph]

- Charged Fermion Sector (7 parameters)

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u \qquad M_d,\ M_e^T = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & (1,-3)c & 0 \\ b & b & 1 \end{pmatrix} y_b v_d \phi_0$$

V_{cb} V_{ub}

$$\boxed{\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},} \Rightarrow \boxed{\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c}$$

Georgi-Jarlskog relations $\Rightarrow V_{d,L} \neq 1$
SU(5) $\Rightarrow M_d = (M_e)^T$
$\Rightarrow$ corrections to TBM related to θ_c

- Neutrino Sector (2 parameters)

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2}}$$

CGs of SU(5) & T′

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta}$$

neutrino mixing angle

1/2

quark mixing angle

complex CGs: leptonic Dirac CPV
(the only non-zero leptonic CPV phase)

prediction for Majorana phases: 0, π

$\Rightarrow$ connection between leptogenesis & CPV in neutrino oscillation

correction accounts for discrepancy between exp best fit value and TBM prediction for solar angle

Numerical Results

- Experimentally: $m_u : m_c : m_t \quad = \quad \theta_c^{7.5} : \theta_c^{3.7} : 1 \qquad m_d : m_s : m_b \quad = \quad \theta_c^{4.6} : \theta_c^{2.7} : 1$
- Model Parameters:

7 parameters in charged fermion sector

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g + (1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}$$

$$\begin{aligned} b &\equiv \phi_0 \psi_0' / \zeta_0 = 0.00304 \\ c &\equiv \psi_0 \zeta_0' / \zeta_0 = -0.0172 \\ k &\equiv y' \psi_0 \zeta_0 = -0.0266 \\ h &\equiv \phi_0^2 = 0.00426 \\ g &\equiv \phi_0'^3 = 1.45 \times 10^{-5} \end{aligned}$$

$$y_t = 1.25$$

$$y_b \phi_0 \zeta_0 \simeq m_b / m_t \simeq 0.011$$

predicting: 9 masses, 3 mixing angles, 1 CP Phase; all agree with exp within 3σ

- CKM Matrix and Quark CPV measures:

CPV entirely from CG coefficients

$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.227 & 0.00412 \\ 0.227 & 0.973 & 0.0412 \\ 0.00718 & 0.0408 & 0.999 \end{pmatrix}$$

$$\begin{aligned} A &= 0.798 \\ \overline{\rho} &= 0.299 \\ \overline{\eta} &= 0.306 \end{aligned}$$

$$\begin{aligned} \beta &\equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 23.6^o, \ \sin 2\beta = 0.734 \ , \\ \alpha &\equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 110^o \ , \\ \gamma &\equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 45.6^o \ , \\ J &\equiv \mathrm{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 2.69 \times 10^{-5} \ , \end{aligned}$$

Direct measurements @ 3σ (ICHEP2010)

$$\begin{aligned} \sin 2\beta &= 0.672^{+0.069}_{-0.07} \\ \gamma\,(\mathrm{deg}) &= 71^{+46}_{-45} \\ \alpha\,(\mathrm{deg}) &= 89^{+21}_{-13} \end{aligned}$$

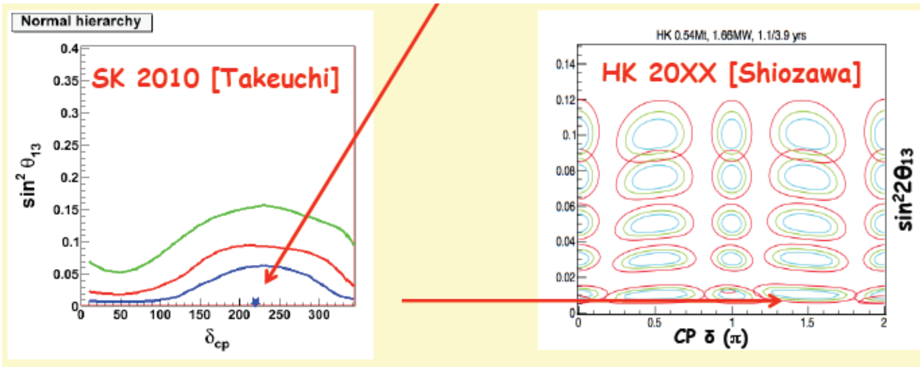
Numerical Results

- Diagonalization matrix for charged leptons $\begin{pmatrix} 0.997e^{i177^o} & 0.0823e^{i131^o} & 1.31\times 10^{-5}e^{-i45^o} \\ 0.0823e^{i41.8^o} & 0.997e^{i176^o} & 0.000149e^{-i3.58^o} \\ 1.14\times 10^{-6} & 0.000149 & 1 \end{pmatrix}$

- MNS Matrix   Note that these predictions do NOT depend on η_0 and ξ_0

$$|U_{MNS}| = \begin{pmatrix} 0.838 & 0.542 & 0.0583 \\ 0.362 & 0.610 & 0.705 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$

prediction for Dirac CP phase: δ = 227 degrees



$$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_\odot = 0.419, \quad |U_{e3}| = 0.0583$$

$$J_\ell = -0.00967$$

Dirac phase the only non-vanishing leptonic CPV phase

$\Rightarrow$ connection between leptogenesis & CPV in neutrino oscillation

SuperK best fit: δ = 220 degrees

- Neutrino Masses: using best fit values for Δm^2

$$\xi_0 = -0.0791\,, \quad \eta_0 = 0.1707\,, \qquad S_0 = 10^{12}\ \text{GeV}$$

2 independent parameters in neutrino sector

$$|m_1| = 0.00134\ \text{eV}, \ |m_2| = 0.00882\ \text{eV}, \ |m_3| = 0.0504\ \text{eV}$$

predicting: 3 masses, 3 angles, 3 CP Phases; both θ_{sol} & θ_{atm} agree with exp

- Majorana phases: $\alpha_{21} = \pi \qquad \alpha_{31} = 0.$

Neutrino Mass Sum Rule

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Three effective neutrino masses determined by two parameters:

$$
\begin{aligned}
m_1 &= (3\xi_0 + \eta_0)^2 \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0} \\
m_2 &= \eta_0^2 \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0} \\
m_3 &= -(-3\xi_0 + \eta_0)^2 \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0}
\end{aligned}
$$

- One sum rule among three neutrino masses:

$$
\begin{aligned}
\left| |\sqrt{m_1}| + |\sqrt{m_3}| \right| &= 2|\sqrt{m_2}| \quad \text{for} \quad (3\xi_0 + \eta_0)(3\xi_0 - \eta_0) > 0 \\
\left| |\sqrt{m_1}| - |\sqrt{m_3}| \right| &= 2|\sqrt{m_2}| \quad \text{for} \quad (3\xi_0 + \eta_0)(3\xi_0 - \eta_0) < 0
\end{aligned}
$$

normal hierarchy predicted

$$
m_2^2 - m_1^2 = (\eta_0^4 - (3\xi_0 + \eta_0)^4) \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0} > 0
$$

$$
m_3^2 - m_1^2 = -24\eta_0 \xi_0 (9\xi_0^2 + \eta_0^2) \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0}
$$

Leptogenesis

M.-C.C, Mahanthappa, arXiv:1107.3856

- TBM from broken discrete symmetries through type-I seesaw   E. Jenkins, A. Manohar, 2008
- exact TBM: $\sin\theta_{13} = 0 \;\Rightarrow\; J_{CP}^{lep} \propto \sin\theta_{13} = 0$ CP violation through Majorana phases: $\alpha_{21},\ \alpha_{31}$
  - no leptogenesis as $\mathrm{Im}(hh^\dagger) = 0$
  - true even when flavor effects included

In usual seesaw realization:
R = diagonal $\Rightarrow$ $\varepsilon_{i\alpha} = 0$

- Asymmetry associated with each flavor α due to N_i decay (vertex correction)

$$\epsilon_{i\alpha} = -\frac{3M_i}{16\pi v^2}\frac{\mathrm{Im}(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{i\beta} R_{i\rho})}{\sum_\beta m_\beta |R_{i\beta}|^2}$$

where $R \;=\; vM^{-1/2} h U_{\mathrm{MNS}} m^{-1/2}$

$$\begin{aligned} h &: \text{ Dirac Yukawa in } M_e,\, M_{RR} \text{ diagonal basis} \\ M &= \mathrm{diag}(M_1, M_2, M_3), \quad \text{RH neutrino absolute masses} \\ m &= \mathrm{diag}(m_1, m_2, m_3), \quad \text{light neutrino absolute masses} \end{aligned}$$

- conditions to have non-zero asymmetry:
  - no flavor effects: R matrix = complex, non-diagonal
  - with flavor effects: R matrix = non-diagonal

Leptogenesis

M.-C.C, Mahanthappa, arXiv:1107.3856

- SUSY SU(5) x T′ model:
  - alternative seesaw + corrections to TBM from charged lepton sector

$$R = vM^{-1/2}U_{\nu,R}M_D U_{\mathrm{TBM}} m^{-1/2} \to \text{real, non-diagonal (12) block}$$

$$R = \begin{pmatrix} -0.816 & 0.577 & 0 \\ 0.577 & 0.816 & 0 \\ 0 & 0 & i \end{pmatrix}$$

- three degenerate RH neutrinos: asymmetry = 0
- RG corrections $\Rightarrow$ small mass splitting

$\Rightarrow$ near degenerate RH masses: resonant enhancement

Leptogenesis

M.-C.C, Mahanthappa, arXiv:1107.3856

- with flavor effects, complex phase in MNS matrix (self-energy diagram):

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma\left(N_i \to l_\alpha + H^c\right) - \Gamma\left(N_i \to l_\alpha^c + H\right)}{\sum_\alpha \left[\Gamma\left(N_i \to l_\alpha + H^c\right) + \Gamma\left(N_i \to l_\alpha^c + H\right)\right]} = -\sum_{j\neq i} \frac{\Gamma_j}{M_j} S_{ij} I_{ij}^\alpha$$

$$\Gamma_j = \frac{1}{8\pi}(hh^\dagger)_{jj} M_j \qquad S_{ij} = \frac{M_i M_j \Delta M_{ij}^2}{(\Delta M_{ij}^2)^2 + M_i^2 \Gamma_j^2} \qquad \Delta M_{ij}^2 = M_j^2 - M_i^2$$

$$\delta_{ij}^R \equiv \frac{M_j}{M_i} - 1$$

$$\delta_{ij}^R = 2(\hat{H}_{ii} - \hat{H}_{jj})t\,, \quad t \equiv \frac{1}{16\pi^2} \ln\left(\frac{\Lambda}{M}\right) \qquad \hat{H} = V^T (h_D h_D^\dagger) V, \text{ and } V = U_{\mathrm{TBM}}$$

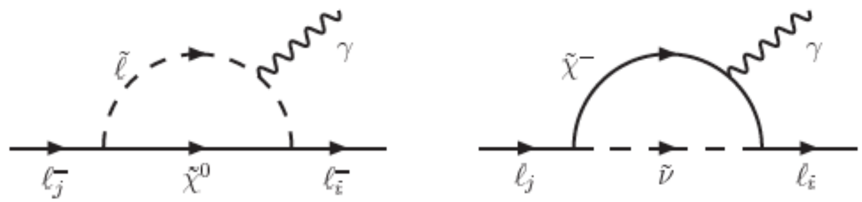
$$I_{ij}^\alpha = \frac{1}{(hh^\dagger)_{ii}(hh^\dagger)_{jj}} \frac{M_i M_j}{v_u^4} \sum_\ell (R_{i\ell} R_{j\ell} m_\ell) \sum_{t,s} \sqrt{m_t m_s} R_{it} R_{js} \mathrm{Im}(U_{\alpha s} U_{\alpha t}^*)$$

- sufficient amount of leptogenesis can be generated $\epsilon_1^\tau \sim -9.04 \times 10^{-7}$
- Dirac phase the only non-vanishing leptonic CPV phase

  ⇒ connection between leptogenesis & low energy CPV

Predictions for LFV Radiative Decay

- SUSY GUTs: slepton-neutralino and sneutrino-chargino loop: Borzumati, Masiero (1986)



- CMSSM: at M_{GUT}, slepton mass matrices flavor blind
- RG evolution: generate off diagonal elements in slepton mass matrices
- dominant contribution: LL slepton mass matrix Hisano, Moroi, Tobe, Yamaguichi (1995)

$$BRji = \frac{\alpha^3}{G_F^2 m_s^8}|(m_{LL}^2)_{ji}|^2 \tan^2\beta$$

$$(m_{LL}^2)_{ji} = -\frac{1}{8\pi^2} m_0^2 (3 + A_0^2/m_0^2) Y_{jk}^\dagger \log\left(\frac{M_G}{M_k}\right) Y_{ki}$$

very model dependent

good approximation to full evolution effects:

$$m_s^8 \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$$

Petcov, Profumo, Takanishi, Yaguna (2003)

Predictions for LFV Radiative Decay

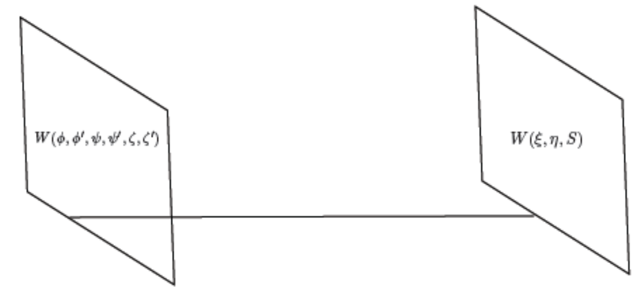
- in SUSY SU(5) x T′ model: M.-C.C, Mahanthappa, Meroni, Petcov under preparation
  - degenerate RH masses
  - ratios of branching fractions depend on mixing & light neutrino masses

$$Y^{+}Y = \begin{pmatrix} 0.000122635 & 0.0000589172 & 0.000131458 \\ 0.0000589172 & 0.000941119 & 0.000720549 \\ 0.000131458 & 0.000720549 & 0.000936627 \end{pmatrix}$$

  - predicting $Br(\mu \to e\gamma) < Br(\tau \to e\gamma) < Br(\tau \to \mu\gamma)$
  - m_0 = 50 GeV, $M_{1/2}$ = 200 GeV, $A_0 = 7m_0$:
    - Br($\tau \to \mu + \gamma$) =1.38E-9
    - Br($\tau \to e + \gamma$) = 4.59E-11
    - Br($\mu \to e + \gamma$) = 9.23E-12

Vacuum Alignment

M.-C.C., Mahanthappa, under preparation



- Z_{12} x Z_{12}' symmetry: too restrictive
  - resort to extra dimensions (5D)
  - in the bulk: Z_{12} x Z_{12}' symmetric
  - on the boundary branes: Z_{12} x Z_{12}' explicitly broken
- Neutrino sector:
  - invariants: $B_1^\nu = \xi^2,\ B_2^\nu = \eta^2,\ T_1^\nu = \xi^3,\ T_2^\nu = \xi^2\eta,\ T_3^\nu = \eta^3 \quad B_3^\nu = S^2\ ,$
  - superpotential: $T_4^\nu = S^3\ ,\quad T_5^\nu = \xi^2 S\ ,\quad T_6^\nu = \eta^2 S\ ,\quad T_7^\nu = \eta S^2$

$$\mathcal{W}_\nu^{flavon} = \sum_i m_i' B_i + \sum_j p_j' T^j$$

  - supersymmetric minimal:

$$\begin{aligned}
F_{\xi_1} &= F_{\xi_2} = F_{\xi_3} = 2(m_1' + p_5 s_0 + p_2 \eta_0) v = 0 \\
F_\eta &= p_7 s_0^2 + 2(m_2' + m_3')\eta_0 + 2 p_6 \eta_0 s_0 + 3 p_3 \eta_0^2 + 3 p_2 v^2 = 0 \\
F_s &= 3 p_4 s_0^2 + 2 p_7 \eta_0 s_0 + p_6 \eta_0^2 + 3 p_5 v^2 = 0
\end{aligned}$$

$$\langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\langle \eta \rangle = \eta_0 \Lambda \qquad \langle S \rangle = S_0$$

Vacuum Alignment

- charged fermion sector:
  - invariants

$$B_1 = \phi^2, \quad B_2 = \phi'^2, \quad B_3 = \phi\phi', \quad B_4 = \zeta N$$

$$T_1 = \phi^3, \quad T_2 = \phi'^3, \quad T_3 = \phi^2\phi', \quad T_4 = \phi'^2\phi, \quad T_5 = N^3, \quad T_6 = \zeta^3, \quad T_7 = \phi^2\zeta$$
$$T_8 = \phi'^2\zeta, \quad T_9 = \phi\phi'\zeta, \quad T_{10} = \phi^2 N, \quad T_{11} = \phi'^2 N, \quad T_{12} = \phi\phi' N, \quad T_{13} = \psi'^2\phi$$
$$T_{14} = \psi'^2\phi', \quad T_{15} = \psi^2\phi, \quad T_{16} = \psi^2\phi', \quad T_{17} = \psi\psi'\phi, \quad T_{18} = \psi\psi'\phi', \quad T_{19} = \psi\psi'\zeta$$

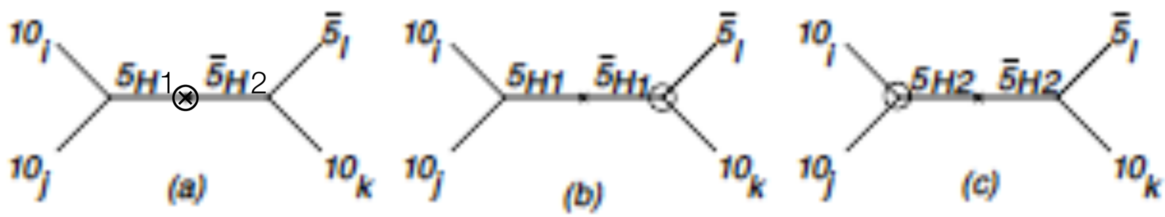
  - superpotential

$$\mathcal{W}_c^{flavon} = \sum_i m_i'' B_i + \sum_i \mu_j'' T^j$$

  - Supersymmetric minima: exist parameter space that satisfy minimization conditions (F=0)

Proton Decay in SU(5) x T´ Model

- proton decay mediated by color triplet Higgsinos (dim-5 operators)
  - generally gives too fast decay rate
  - Z_{12} x Z_{12} forbid (vertices in circles)



  - no Higgsino mediated proton decay
  - Planck induced operators: Yukawa suppressed
- proton decay mediated by gauge boson (dim-6 operators)
  - non-minimal Higgs content, model prediction is within current experimental limits

Curing FCNC Problem: Family Symmetry vs MFV

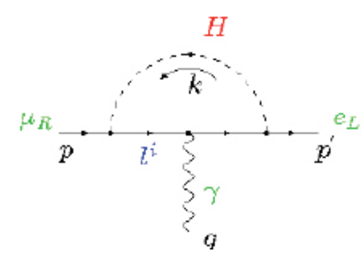
- low scale new physics severely constrained by flavor violation
- Minimal Flavor Violation

  D'Ambrosio, Giudice, Isidori, Strumia (2002);
  Cirigliano, Grinstein, Isidori, Wise (2005)

  - assume Yukawa couplings the only source of flavor violation

- Example: Warped Extra Dimension

  - wave function overlap ⇒ naturally small Dirac mass $\psi_{(0)} \sim e^{(1/2-c)ky}$
  - non-universal bulk mass terms (c) ⇒ FCNCs at tree level ⇒ Λ > O(10) TeV
    - FCNCs: present even in the limit of massless neutrinos
      - tree-level: μ-e conversion, μ→3e, etc
    - charged current
      - one-loop: μ→e+γ, τ→e+γ, τ→μ+γ



  - fine-tuning to get large mixing and mild mass hierarchy for neutrinos

Curing FCNC Problem: Family Symmetry vs MFV

- Two approaches:
  - Minimal Flavor Violation in RS

    quark sector: A. Fitzpatrick, G. Perez, L. Randall (2007)
    lepton sector: M.-C.C., H.B. Yu (2008)

$$C_e = aY_e^\dagger Y_e, \quad C_N = dY_\nu^\dagger Y_\nu, \quad C_L = c(\xi Y_\nu Y_\nu^\dagger + Y_e Y_e^\dagger)$$

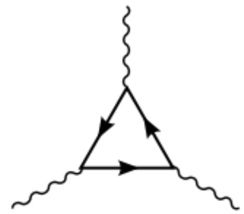
  - T´ symmetry in the bulk for quarks & leptons:

    M.-C.C., K.T. Mahanthappa, F. Yu (PLB2009);
    A4 for leptons: Csaki, Delaunay, Grojean, Grossmann

    - TBM neutrino mixing: common bulk mass term, no tree-level FCNCs
    - TBM mixing and masses decouple: no fine-tuning
    - realistic masses and mixing angles in quark sector
    - no tree-level FCNCs in lepton sector and 1-2 family of quark sector

- Family Symmetry: alternative to MFV to avoid FCNCs in TeV scale new physics
  - many family symmetries violate MFV ⇒ possible new FV contributions

Anomalous vs Non-anomalous U(1)´



- anomaly cancellations: relating charges of different fermions
  - $[U(1)']^3$ condition generally difficult to solve
- most models utilized anomalous U(1)´:
  - mixed anomaly: cancelled by Green-Schwarz mechanism
  - $[U(1)']^3$ anomaly: cancelled by exotic fields besides RH neutrinos
  - U(1)´ broken at fundamental string scale
  - earlier claim that U(1)´ has to be anomalous to be compatible with SU(5) while giving rise to realistic fermion mass and mixing patterns Ibanez, Ross, 1994

constraints not as stringent

- non-anomalous U(1)´ can be compatible with SUSY SU(5) while giving rise to realistic fermion mass and mixing patterns M.-C.C, D.R.T. Jones, A. Rajaraman, H.B. Yu, 2008
  - no exotics other than 3 RH neutrinos
  - U(1)´ also forbids Higgs-mediated proton decay
- can be utilized to get TeV seesaw for neutrino masses M.-C.C, de Gouvea, Dobrescu, 2006

Non-anomalous U(1)′ in AMSB

M.-C. C., J.-R. Huang, arXiv:1011.0407

- AMSB: all sfermion masses depend on $m_{3/2}$ + low energy dynamics

$$(m^2)^i_j = \frac{1}{2} m^2_{3/2} \mu \frac{d}{d\mu} \gamma^i_j$$

RG invariant

- Predict tachyonic slepton masses!
- RG invariant solution to the tachyonic slepton mass problem in AMSB

I. Jack, D.R.T. Jones, 1999

$$\bar{m}^2_L = m^2_L + \zeta q_{L_i} \delta^i_j$$

ζ: FI D-term contributions

pure AMSB contributions

- All mixed anomalies = 0 ⇒ Green-Schwarz can't work

  ⇒ non-anomalous U(1)′

- generation dependent U(1)′ charges ⇒ fermion masses and mixing angles
- predict testable mass sum rules among sparticles at colliders

Non-universal, Non-anomalous U(1)´ Model

M.-C. C., J.-R. Huang, arXiv:1011.0407

- sparticle masses: pure AMSB contributions + D-term contributions

$$\begin{aligned} \bar{m}_Q^2 &= m_Q^2 + \zeta q_{Q_i} \delta_j^i, \\ \bar{m}_{u^c}^2 &= m_{u^c}^2 + \zeta q_{u_i} \delta_j^i, \\ \bar{m}_{d^c}^2 &= m_{d^c}^2 + \zeta q_{d_i} \delta_j^i, \end{aligned} \qquad \begin{aligned} \bar{m}_L^2 &= m_L^2 + \zeta q_{L_i} \delta_j^i, \\ \bar{m}_e^2 &= m_e^2 + \zeta q_{e_i} \delta_j^i, \end{aligned} \qquad \begin{aligned} \bar{m}_{H_u}^2 &= m_{H_u}^2 + \zeta q_{H_u}, \\ \bar{m}_{H_d}^2 &= m_{H_d}^2 + \zeta q_{H_d}, \end{aligned}$$

- search for charges that satisfy:
  - all 6 anomaly cancellation conditions
  - realistic quark masses (6), charged lepton masses (3), neutrino masses and mixing angles (6)
  - electroweak symmetry breaking
  - all squark and slepton masses^2 positive

Resulting Yukawa Sector

- Charged fermion sector:

$$Y_u \sim \begin{pmatrix} \lambda^{|q_{Q_1}+q_{u_1}+q_{H_u}|} & \lambda^{|q_{Q_1}+q_{u_2}+q_{H_u}|} & \lambda^{|q_{Q_1}+q_{u_3}+q_{H_u}|} \\ \lambda^{|q_{Q_2}+q_{u_1}+q_{H_u}|} & \lambda^{|q_{Q_2}+q_{u_2}+q_{H_u}|} & \lambda^{|q_{Q_2}+q_{u_3}+q_{H_u}|} \\ \lambda^{|q_{Q_3}+q_{u_1}+q_{H_u}|} & \lambda^{|q_{Q_3}+q_{u_2}+q_{H_u}|} & \lambda^{|q_{Q_3}+q_{u_3}+q_{H_u}|} \end{pmatrix} \sim \begin{pmatrix} \lambda^{10} & \lambda^{|\frac{7}{2}-\frac{2a'}{5}|} & \lambda^{|\frac{13}{2}+\frac{8a'}{5}|} \\ \lambda^{|\frac{7}{2}+\frac{2a'}{5}|} & \lambda^{|-3|} & \lambda^{|2a'|} \\ \lambda^{|\frac{7}{2}-\frac{8a'}{5}|} & \lambda^{|-3-2a'|} & \lambda^{0} \end{pmatrix}$$

$$Y_d \sim \begin{pmatrix} \lambda^{|q_{Q_1}+q_{d_1}+q_{H_d}|} & \lambda^{|q_{Q_1}+q_{d_2}+q_{H_d}|} & \lambda^{|q_{Q_1}+q_{d_3}+q_{H_d}|} \\ \lambda^{|q_{Q_2}+q_{d_1}+q_{H_d}|} & \lambda^{|q_{Q_2}+q_{d_2}+q_{H_d}|} & \lambda^{|q_{Q_2}+q_{d_3}+q_{H_d}|} \\ \lambda^{|q_{Q_3}+q_{d_1}+q_{H_d}|} & \lambda^{|q_{Q_3}+q_{d_2}+q_{H_d}|} & \lambda^{|q_{Q_3}+q_{d_3}+q_{H_d}|} \end{pmatrix} \sim \begin{pmatrix} \lambda^{5} & \lambda^{|\frac{19}{2}-\frac{2a'}{5}|} & \lambda^{|\frac{15}{2}+\frac{8a'}{5}|} \\ \lambda^{|-\frac{3}{2}+\frac{2a'}{5}|} & \lambda^{3} & \lambda^{|1+2a'|} \\ \lambda^{|-\frac{3}{2}-\frac{8a'}{5}|} & \lambda^{|3-2a'|} & \lambda^{1} \end{pmatrix}$$

$$Y_e \sim \begin{pmatrix} \lambda^{|q_{L_1}+q_{e_1}+q_{H_d}|} & \lambda^{|q_{L_1}+q_{e_2}+q_{H_d}|} & \lambda^{|q_{L_1}+q_{e_3}+q_{H_d}|} \\ \lambda^{|q_{L_2}+q_{e_1}+q_{H_d}|} & \lambda^{|q_{L_2}+q_{e_2}+q_{H_d}|} & \lambda^{|q_{L_2}+q_{e_3}+q_{H_d}|} \\ \lambda^{|q_{L_3}+q_{e_1}+q_{H_d}|} & \lambda^{|q_{L_3}+q_{e_2}+q_{H_d}|} & \lambda^{|q_{L_3}+q_{e_3}+q_{H_d}|} \end{pmatrix} \sim \begin{pmatrix} \lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{4} & \lambda^{3} & \lambda^{1} \\ \lambda^{4} & \lambda^{3} & \lambda^{1} \end{pmatrix}$$

Non-integer powers:
naturally give rise to texture zeros
in Yukawa matrices
(# of flavon fields inserted
must be integer)

Resulting Yukawa Sector

- Neutrino sector:

$$Y_\nu \sim \begin{pmatrix} \lambda^{|q_{L_1}+q_{N_1}+q_{H_u}|} & \lambda^{|q_{L_1}+q_{N_2}+q_{H_u}|} & \lambda^{|q_{L_1}+q_{N_3}+q_{H_u}|} \\ \lambda^{|q_{L_2}+q_{N_1}+q_{H_u}|} & \lambda^{|q_{L_2}+q_{N_2}+q_{H_u}|} & \lambda^{|q_{L_2}+q_{N_3}+q_{H_u}|} \\ \lambda^{|q_{L_3}+q_{N_1}+q_{H_u}|} & \lambda^{|q_{L_3}+q_{N_2}+q_{H_u}|} & \lambda^{|q_{L_3}+q_{N_3}+q_{H_u}|} \end{pmatrix} \sim \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$$Y_N \sim \begin{pmatrix} \lambda^{|2q_{N_1}|} & \lambda^{|q_{N_1}+q_{N_2}|} & \lambda^{|q_{N_1}+q_{N_3}|} \\ \lambda^{|q_{N_2}+q_{N_1}|} & \lambda^{|2q_{N_2}|} & \lambda^{|q_{N_2}+q_{N_3}|} \\ \lambda^{|q_{N_3}+q_{N_1}|} & \lambda^{|q_{N_3}+q_{N_2}|} & \lambda^{|2q_{N_3}|} \end{pmatrix} . \quad Y_N \langle\Psi\rangle \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \end{pmatrix} \langle\Psi\rangle$$

  - seesaw mechanism ⇒ effective neutrino mass matrix

$$m_\nu \sim Y_\nu Y_N^{-1} Y_\nu^T \frac{v^2}{\langle\Psi\rangle} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \frac{v^2}{\langle\Psi\rangle}$$

2 large & 1 small mixing angles:
Δm^2_{atm} and Δm^2_{sol} agree w/ exp

Anomaly-free Charges

M.-C. C., J.-R. Huang, arXiv:1011.0407

- two parameters: $\boxed{a' \text{ and } q_{L_3}}$
- parametrizing charges that satisfy all anomaly cancelation conditions + fermion mass and mixing angle constraints

Field	$U(1)'_{\rm NAF}$ charge
L_1	$q_{L_1} = 1 + q_{L_3}$
L_2	$q_{L_2} = q_{L_3}$
L_3	$q_{L_3} = q_{L_3}$
e_1^c	$q_{e_1} = -(-386375 + 65664a'^2 + 153000q_{L_3} + 1080a'(37 + 48q_{L_3}))/(180(425 + 144a'))$
e_2^c	$q_{e_2} = -(-309875 + 65664a'^2 + 153000q_{L_3} + 1080a'(61 + 48q_{L_3}))/(180(425 + 144a'))$
e_3^c	$q_{e_3} = -(-156875 + 65664a'^2 + 153000q_{L_3} + 1080a'(109 + 48q_{L_3}))/(180(425 + 144a'))$
Q_1	$q_{Q_1} = 38/9 + 2a'/5 - q_{L_3}/3$
Q_2	$q_{Q_2} = -41/18 + 4a'/5 - q_{L_3}/3$
Q_3	$q_{Q_3} = (-205 - 108a' - 30q_{L_3})/90$
u_1^c	$q_{u_1} = (55296a'^2 + 720a'(173 + 48q_{L_3}) + 125(-371 + 816q_{L_3}))/(180(425 + 144a'))$
u_2^c	$q_{u_2} = (44928a'^2 + 1080a'(-69 + 32q_{L_3}) + 125(-4349 + 816q_{L_3}))/(180(425 + 144a'))$
u_3^c	$q_{u_3} = (96768a'^2 + 720a'(217 + 48q_{L_3}) + 125(-2513 + 816q_{L_3}))/(180(425 + 144a'))$
d_1^c	$q_{d_1} = -(-46625 + 25344a'^2 + 17000q_{L_3} + 480a'(107 + 12q_{L_3}))/(60(425 + 144a'))$
d_2^c	$q_{d_2} = (32275 - 5760a'^2 - 3400q_{L_3} - 72a'(63 + 16q_{L_3}))/(5100 + 1728a')$
d_3^c	$q_{d_3} = (22075 - 2304a'^2 - 3400q_{L_3} - 96a'(-23 + 12q_{L_3}))/(5100 + 1728a')$
N_1	$q_{N_1} = (-335375 + 57240a' + 65664a'^2)/(180(425 + 144a'))$
N_2	$q_{N_2} = (-335375 + 57240a' + 65664a'^2)/(180(425 + 144a'))$
N_3	$q_{N_3} = (-335375 + 57240a' + 65664a'^2)/(180(425 + 144a'))$
H_u	$q_{H_u} = -(-488375 + 65664a'^2 + 76500q_{L3} + 1080a'(5 + 24q_{L3}))/(180(425 + 144a'))$
H_d	$q_{H_d} = (65664a'^2 + 1080a'(133 + 24q_{L3}) + 125(-643 + 612q_{L3}))/(180(425 + 144a'))$
Φ	$q_\Phi = -1/3$
Ψ	$q_\Psi = (182375 - 109080a' - 65664a'^2)/(38250 + 12960a')$

A Solution

- neutralino LSP

$$\zeta = 1.7 \times (100GeV)^2$$

Field	h_0	H_0	A_0	H^+	$\tilde{g}$	χ_1	χ_2	χ_3	χ_4	$\chi_1^\pm$	$\chi_2^\pm$	$\tilde{u}_L$	$\tilde{u}_R$	$\tilde{d}_L$	$\tilde{d}_R$	$\tilde{c}_L$
Mass (GeV)	114	163	162	181	880	134	361	489	498	134	496	825	790	829	979	731
Field	$\tilde{c}_R$	$\tilde{s}_L$	$\tilde{s}_R$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\mu}_L$	$\tilde{\mu}_R$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_{eL}$	$\tilde{\nu}_{\mu L}$	$\tilde{\nu}_{\tau L}$
Mass (GeV)	742	735	1035	321	782	748	915	348	273	322	240	143	322	338	312	310

TABLE I: The mass spectrum of the sparticles, with $a' = -27/5$ and $q_{L_3} = 1/2$.

Non-anomalous U(1)′ in AMSB

- predict testable (RG invariant) mass sum rules among sparticles at colliders

$$\bar{m}^2_{Q_i} + \bar{m}^2_{u^c_i} + \bar{m}^2_{H_u} = (m^2_{Q_i} + m^2_{u^c_i} + m^2_{H_u})_{AMSB}\ (i = 1,2,3)$$

$$\bar{m}^2_{Q_i} + \bar{m}^2_{d^c_i} + \bar{m}^2_{H_d} = (m^2_{Q_i} + m^2_{d^c_i} + m^2_{H_d})_{AMSB}\ (i = 1,2,3)$$

$$\bar{m}^2_{L_i} + \bar{m}^2_{e^c_i} + \bar{m}^2_{H_d} = (m^2_{L_i} + m^2_{e^c_i} + m^2_{H_d})_{AMSB}\ (i = 1,2,3)$$

$\rightarrow$ f(g,Y) x $(m_{3/2})^2$

$$m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} + m^2_{\tilde{d}_L} + m^2_{\tilde{d}_R} + m^2_{\tilde{c}_L} + m^2_{\tilde{c}_R} + m^2_{\tilde{s}_L} + m^2_{\tilde{s}_R} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} + m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2}$$
$$= 2\sum_{i=1}^{3}(2m^2_{\tilde{Q}_i} + m_{\tilde{u}^c_i} + m_{\tilde{d}^c_i}) + 2\sum_{i=1}^{3}(m^2_{u_i} + m^2_{d_i})\,,$$

$$m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} + m^2_{\tilde{\mu}_L} + m^2_{\tilde{\mu}_R} + m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} + m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} + m^2_{\tilde{c}_L} + m^2_{\tilde{c}_R} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}$$
$$= \sum_{i=1}^{3}(m^2_{\tilde{L}_i} + m^2_{\tilde{e}^c_i} + m^2_{\tilde{Q}_i} + m^2_{\tilde{u}^c_i}) + 2\sum_{i=1}^{3}(m^2_{e_i} + m^2_{u_i})\,.$$

Flavor Physics at the Collider

Summary

- SUSY SU(5) x T′ symmetry: tri-bimaximal lepton mixing & realistic CKM matrix
- complex CG coefficients in T′: origin of CPV both in quark and lepton sectors
- Z_{12} x Z_{12}': only 9 parameters in Yukawa sector
  - dynamical origin of mass hierarchy (including m_b vs m_t)
  - forbid Higgsino-mediated proton decay

quark CP phase: γ = 45.6 degrees

- interesting sum rules: $\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05}$

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta}$$

right amount to account for discrepancy between exp best fit value and TBM prediction

leptonic Dirac CP phase: δ = 227 degrees
(SuperK best fit: 220 degrees)

- Leptonic Dirac CP phase:
  - sufficient amount of lepton number asymmetry
  - the only non-vanishing leptonic CPV phase connection between leptogenesis and low energy CPV
- SUSY - Flavor Complementarity